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Controlled Decoherence of Three-level Floating Qubit

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Abstract A scheme is proposed to controll decoherence of three-level floating qubit by designing an external electric circuit with superconductive flux qubit. The results show that it may not only raise gate speed but also extend decoherence time for three-level structure.

Keywords Decoherence · Floating qubit · Controll

1 Introduction

With the continuous development of quantum engineering technology, including the improvement of material performance and enhancement of microwave technology, coherent times of qubits have increased from tens of nanoseconds to a few microseconds [1, 2]. In a really physical system, however, the interaction between solid-state qubits and the environment is inevitable, which leads to decoherence [3, 4]. It is well-known that the decoherence in quantum bit circuits is presently a major limitation to their uses for quantum computing purposes [5–7] because it strongly affects the gate speed and gate error rate. In the practical applications, in addition, it is important that a typical gate operation time must be much smaller than the decoherent time. Thus, there exists a fundamental issue of how to increase the decoherence time.

It is known that the multilevel structure of the qubits in the superconducting quantum interference device (SQUID) is advantageous to the exaltation of computing speed of the qubits, however, the intrinsic gate error and leakage states in the computing process cause significant errors even though a simple gate operations for one-bit SQUID [8]. As pointed out by Kis and Paspalakis [9], for the sake of exalting the gate speed of superconductive

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flux qubit, one should adopt a way of the three-level structure in quantum calculation. Based on the conventional configuration of SQUID qubits, furthermore, Zhou et al. proposed a three-level (Λ-type) rf-SQUID qubit [10], where the states of the qubit with the two lower flux states, $|0\rangle$ and $|1\rangle$, and an auxiliary upper state $|e\rangle$ form a Λ-type system. Thus the manipulation of the qubit is done by two microwave fields that couple the lower states ($|0\rangle$ and $|1\rangle$) to upper state $|e\rangle$. As the transition matrix elements, corresponding to the transitions $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$, are larger than that of the other transitions, the three-level SQUID qubit has been shown to be more favourable than the conventional two-level SQUID qubit for implementing a NOT gate, where the auxiliary level $|e\rangle$ is used to significantly increase the speed and reduce errors of quantum gate operations. In the general situations, however, it may be a contradictory issue for the exaltation of both the gate speed and the decoherent time at the same time. In previous work for flux qubit, we not only investigated the impact of asymmetry on quantum gate speed, but also proposed a operating scheme how to both improve the gate speed and extend the decoherent time [11, 12].

In order to overcome the influence of decoherence, one of the tactics proposed by the electrical engineers and researchers is to improve the decoherent time of the qubits by advancing the hardware design of the circuits. Electrically floating qubit may be such an efficient design way. For the floating charge qubit, in two level approximation, the various noisy factors inducing both decoherence and presented corresponding countermeasure were discussed at the reference [13] in detail. However, M. Steffen *et al.* pointed out that the results of charge qubits cannot be simply extended to the flux qubits [14]. For floating flux qubits, capacitive coupling easily occurs in the grounded capacitance and distributed capacitance between lead wires, which result factually in decoherence so as to reduce the decoherent time. In the present work, by combining network graph theory with the Caldeira-Leggett model in the multilevel quantum description method [15–18], the expressions of the relaxation and the decoherent times of the qubit with three-level structure are analyzed in detail. By designing an electric circuit of the floating flux qubit in the regime of three-level structure, furthermore, we proposed a manipulation scheme how to both improve the gate speed and extend the decoherence time at the same time.

2 Controlled Decoherence

In the Hilbert space spanned by the eigenstate $|m\rangle$ with eigenenergy $E_m (m = 1, 2, \ldots)$, the master equation from Hamiltonian of physical system may be obtained in the Markovian approximation, i.e. [19]

$$\dot{\rho}_{nm}(t) = -i\omega_{nm}\rho_{nm}(t) - \sum_{kl} R_{nmkl}\rho_{kl}(t), \tag{1}$$

which is called Redfield equation, while $\rho_{mn} = \langle m|\hat{\rho}|n\rangle$ and $\omega_{nm} = \omega_n - \omega_m$. Here the Redfield relaxation tensor is

$$R_{nmkl} = \frac{1}{2\hbar^2}\left[-x_{lmnk}J(\omega_{nk}) - x_{knml}J(\omega_{ml}) + \delta_{lm}\sum_i x_{niik}J(\omega_{ik}) + \delta_{nk}\sum_i x_{miil}J(\omega_{il})\right], \tag{2}$$

with $x_{lmnk} = \langle l|x|m\rangle\langle n|x|k\rangle$. x is a generalized coordinate of the quantum system and the effect of a thermal bath on a quantum system is characterized by a spectral density $J(\omega)$. In

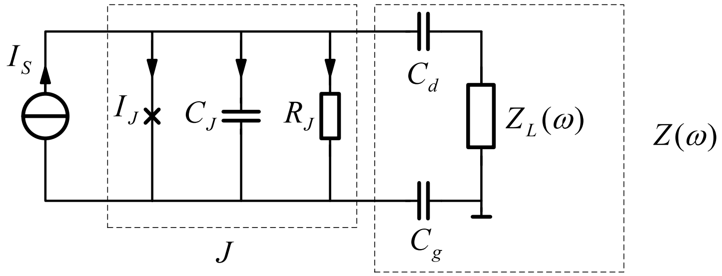

Fig. 1 Circuit of floating qubit with external admittance $Y(\omega)$, where $Y(\omega)$ consists of grounded capacitance that can form capacitance coupling and stray capacitance between lead wires

the semiclassical approximation, the relaxation, decoherence and pure dephasing times of the system may be expressed in terms of the spectral density of the heat bath fluctuations.

It is known that in the two-level structure for the superconducting circuit containing flux qubit, it was proved that noise caused by fluctuations of spectral density can be computed from the frequency-dependent damping coefficients of the quantum Langevin equation with the spin-boson mode. Thus, it is interesting to expand our study for the noise effects in three-level structure by using the Langevin equation, which may be described by a floating qubit circuit as shown in Fig. 1.

In Fig. 1, there are a grounded capacitance C_g and a distributed capacitance C_d in the bias circuit of the flux qubit, which also form to the impedance of the circuit [20], while $Z_L(\omega)$ is an analogical other environment impedance. Using the Kirchhoff law in the qubit circuit described by Fig. 1, the quantum Langevin equation may be expressed as

$$C\Phi_0\frac{d^2x}{dt^2}+\Phi_0\int_{t_0}^{t}d\tau Y(t-\tau)\frac{dx(\tau)}{d\tau}+\frac{1}{\Phi_0}\frac{dV(x)}{dx}=I_S(t), \tag{3}$$

where $\Phi_0\equiv h/2e$ is a unit of superconducting quantum flux. The damping coefficient $Y(t-\tau)$ is an equivalent admittance of the external circuits, $V(x)$ the potential energy, $I_S(t)$ the fluctuation force of undulation of ambient magnetic field. The spectral density of heat bath may be expressed by

$$J(\omega)=\hbar\omega G(\omega)\left[1+\coth\left(\frac{\hbar\omega}{2k_BT}\right)\right], \tag{4}$$

where T is an ambient temperature and $G(\omega)$ is a real part of the frequency-dependent admittance $Y(\omega)$ of the external circuit. The relaxation, pure dephasing and decoherent times of the system may be respectively expressed as [21]

$$\frac{1}{T_1}=\frac{2\Phi_0^2}{\hbar}|x_{jk}|^2J(\omega_{jk}), \tag{5}$$

$$\frac{1}{T_\phi}=\frac{2\Phi_0^2}{\hbar}|x_{jj}-x_{kk}|^2J(\omega)|_{\omega\to 0}, \tag{6}$$

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{1}{T_\phi}, \tag{7}$$

where $x_{jk}=\langle j|x|k\rangle$ is an tunneling amplitude between the two wells.

An actual SQUID has always limited decay impedance as well as decoherence time. At low temperatures, the dephasing time is an inverse with respect to temperature. For an extremely high temperature, furthermore, setting $T_\phi \to \infty$, there is $T_2 = 2T_1$. Differently from the gate speed, the decoherent time will reduce with increasing of coupling matrix elements [22]. Fortunately, one may extend the decoherent time to the regime of three-level structure by controlling the external circuit. Around the low temperature, from (5)–(7), we have

$$T_2 = \frac{\hbar}{\Phi_0^2 |x_{jk}|^2 \omega_{jk} G(\omega_{jk})}, \tag{8}$$

which determines the coupling matrix elements.

Differently from in the two-level structure, in the three-level scheme, one may have two manipulation processes [23]. Firstly, one may let a microwave pulse with frequency ω_a last time t_{0e}. After the pulse disappears at the moment when the transition $|0\rangle \to |e\rangle$ is completed, secondly, a second microwave pulse with frequency ω_b is applied and lasts time t_{e1} for the three-level system to complete the transition $|e\rangle \to |1\rangle$, where the two microwave pulses are of the same amplitude. Thus, the total decoherent time for the three-level system is given by

$$T_{2\Lambda} = \frac{\hbar}{\Phi_0^2 |x_{0e}|^2 \omega_{0e} G(\omega_{0e})} + \frac{\hbar}{\Phi_0^2 |x_{e1}|^2 \omega_{e1} G(\omega_{e1})}. \tag{9}$$

From (8) or (9), we find that if the function,

$$f(\omega) = \omega G(\omega), \tag{10}$$

is a decreasing function of frequency ω, the decoherent time may be effectively increased.

According to the ideas, in order to obtain not only a fast speed of the gate but also a longer decoherence time, an electric circuit is designed according to Figs. 2 and 3, where the mutual-inductor coefficient between primary and secondary circuits is M and $Y_0(\omega) = G_0(\omega) + iB_0(\omega)$ is admittance of secondary circuit. It is worth specially emphasizing that in realistic quantum circuits, the grounded capacitance must be adopted in distributed parameter instead of lumped parameter. According to Fig. 3, we have

$$Y(\omega) = \frac{\omega^2 M^2 G_0 - i(\omega^2 M^2 B_0 + \omega L - \frac{1}{\omega C_{eff}})}{\omega^4 M^4 G_0^2 + (\omega^2 M^2 B_0 + \omega L - \frac{1}{\omega C_{eff}})^2}. \tag{11}$$

Inserting (11) into (8), we obtain

$$T_2 = \frac{\hbar}{\Phi_0^2 |x_{jk}|^2} \frac{\omega_{jk}^4 M^4 G_0^2(\omega_{jk}) + [\omega_{jk} L + \omega_{jk}^2 M^2 B_0(\omega_{jk}) - \omega_{jk}^{-1} C_{eff}^{-1}]^2}{\omega_{jk}^3 M^2 G_0(\omega_{jk})}. \tag{12}$$

In Fig. 3, a particular circuit mode containing the flux qubit is shown to control the frequency-dependent admittance $Z(\omega)$. Adjusting mutual-inductor coefficients between inductors L_1 and L_2 and letting them satisfy the relation $L_1 + L_2 - 2M_{12} = 0$, i.e., $B_0(\omega) = 0$, we can obtain

$$f(\omega) = \frac{\omega^3 M^2 G_0}{\omega^4 M^4 G_0^2 + (\omega L - \frac{1}{\omega C_{eff}})^2}. \tag{13}$$

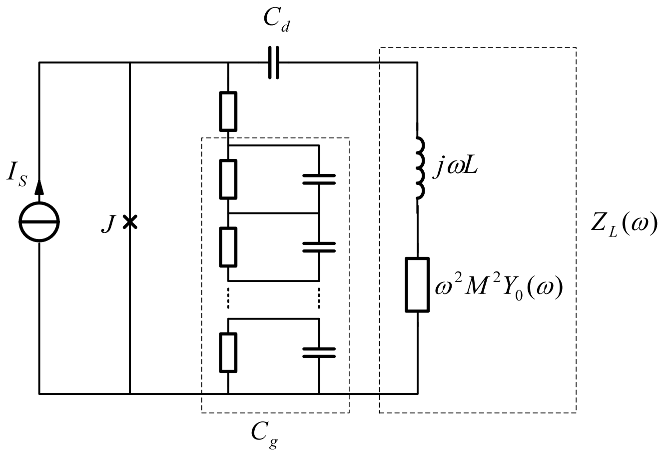

Fig. 2 A simple circuit model is shown with floating flux qubit, where the *left* is circuit of qubit and the *right* is the circuit that controls qubit by coupling mutual-inductor. Note that the grouded capacitance is distributed

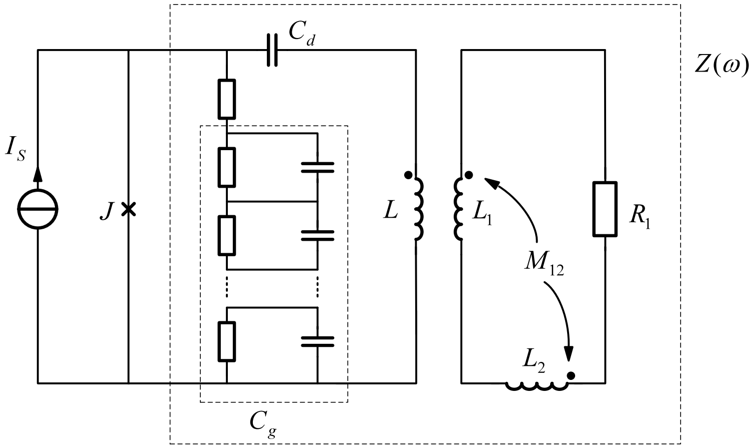

Fig. 3 A particular circuit model is showed with flux qubit, where mutual-inductor coefficient between primary and secondary circuits is M. In secondary circuit, resistor R_1, inductors L_1 and L_2 are in series, and mutual-inductor coefficient between inductors L_1 and L_2 is M_{12}

From (11), we see that in the realistic situation, C_{eff} is a limited value so as to have a higher contribution to the decoherence of quantum bit. If C_{eff} is not infinity, generally, the decoherent time will be decreased.

Next, we further analyze how to improve the flux qubits in reactive environment.

If $\omega^2 LC_{eff} = 1$, the controlling circuit is in resonance state, we have

$$f(\omega) = \frac{R_1}{\omega M^2},$$

which is a decreasing function of frequency ω. From (12) and (8), the decoherent time of the qubit in the resonance state is given by

$$T_2 = \frac{\hbar}{\Phi_0^2 |x_{jk}|^2} \cdot \frac{\omega_{jk} M^2}{R_1}, \tag{14}$$

which indicates that for the resonant state with $\omega^2 L C_{eff} = 1$. Through enhancing the controllable mutual-inductor coefficient between primary and secondary circuits and decreasing the controllable resistance R_1 in the secondary circuit, we may overcome decoherence of the circuit. It is worth noting that extending the decoherent time by adjusting the resistance in the secondary circuit may be a good choice because the manipulation of resistance is very easy.

As an example, the decoherent times are calculated in a realistic SQUID system with the same parameters as Paspalakis et al.'s [22], where $L = 10^2$ pH, $C = 40$ fF, and $I_C = 3.95$ μA. Thus, the values of the two lowest energies and auxiliary level $|4\rangle$ of the system are $\hbar\omega_0 = 7.81984$ meV, $\hbar\omega_2 = 8.00136$ meV, $\hbar\omega_4 = 8.14057$ meV so that the matrix elements are $x_{02} = 0.0323051$, $x_{40} = 0.00539798$, $x_{42} = 0.0428826$. $M = 10^4$ pH and $R_1 = 10^2$ Ω. In resonance state, one has

$$T_2 = 6.5 \text{ ns} \quad \text{and} \quad T_{2\Lambda} = 414 \text{ ns}.$$

If $\omega^2 L C_{eff} \neq 1$, by calculating from the parameters realized in present quantum engineering, we know that $\omega^2 L C_{eff} \ll 1$. From (12) and (8), thus, we have

$$T_2 = \frac{\hbar}{\Phi_0^2 |x_{jk}|^2} \cdot \frac{R_1}{\omega_{jk}^5 M^2 C_{eff}^2}, \tag{15}$$

which means that the grounded capacitance and distributed capacitance of the lead wires in quantum circuit have significantly impact on the decoherent time of the qubit. For this case, the decoherent time is mainly not only determined by the environmental impedance but also depended on the grounded capacitance and distributed capacitance. After a quantity estimate with the same parameters, however, we find that though there exists $T_{2\Lambda} > T_2$ in the nonresonant situation, the decoherent times are as small as for the nonresonant state not to be used to extend the coherent times.

From above analysis, we know that for floating flux qubit, the decoherent time reduces because there exist the effect couplings of between finite grounded capacitance and the environment and between stray capacitance and the environment. By using the resonant states in the three-level system that are resulted from the controllable shunt connecting and series connecting inductive elements in the environmental circuit, fortunately, the decoherent time of the floating flux qubit may be extended and the gate speed may be raised effectively.

3 Conclusion

In summary, although there exists a contradiction between the raising of gate speed and the exalting of decoherence time, it may be conquered by rationally designing electric circuit, where a simple circuit mode containing floating flux qubit and a particular circuit mode containing flux qubit are shown in Figs. 2 and 3. By increasing the controllable mutual-inductor coefficient M and decreasing the controllable resistor R_1 in the floating flux qubit circuits 2 and 3, one may not only raise the gate speed but also extend the decoherence time under a reasonable choice of externally controllable parameters in the electric circuit.

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